Oscillatory neck propagation in polymer films:
A novel model and its refinements

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The generalised Toda's model

Stress-strain relation

$$\dot{\sigma}_0 = \frac{1}{D\left(\chi; L_0\right)} \left\{ V - |\dot{\chi}| \left[\frac{\sigma_0}{E_0} \left(\lambda^2 \frac{E_0}{E_1} - 1 \right) + (\lambda - 1) \right] \right\}$$

Heat balance equation

$$\begin{aligned} S\rho c_p \left(\frac{\partial T}{\partial t} + (1 + H(x)(\lambda - 1))|\dot{\chi}| \frac{\partial T}{\partial x} \right) \\ = -r\left(T - T_a\right) + \frac{\partial}{\partial x} \left(\kappa \frac{\partial T}{\partial x} \right) + S_0 \eta (\lambda - 1) |\dot{\chi}| \sigma_0 \delta(x) \end{aligned}$$

Eyring's equation

$$|\dot{\chi}| = \frac{2v_0}{\lambda - 1} \exp\left[\frac{-\Delta F}{k_B T_0} \right] \sinh\left[\frac{\alpha \sigma_0}{k_B T_0} \right] =: \mathcal{E}(\sigma, \tau_0)$$

Parameters & Interpretations

Parameters we tune

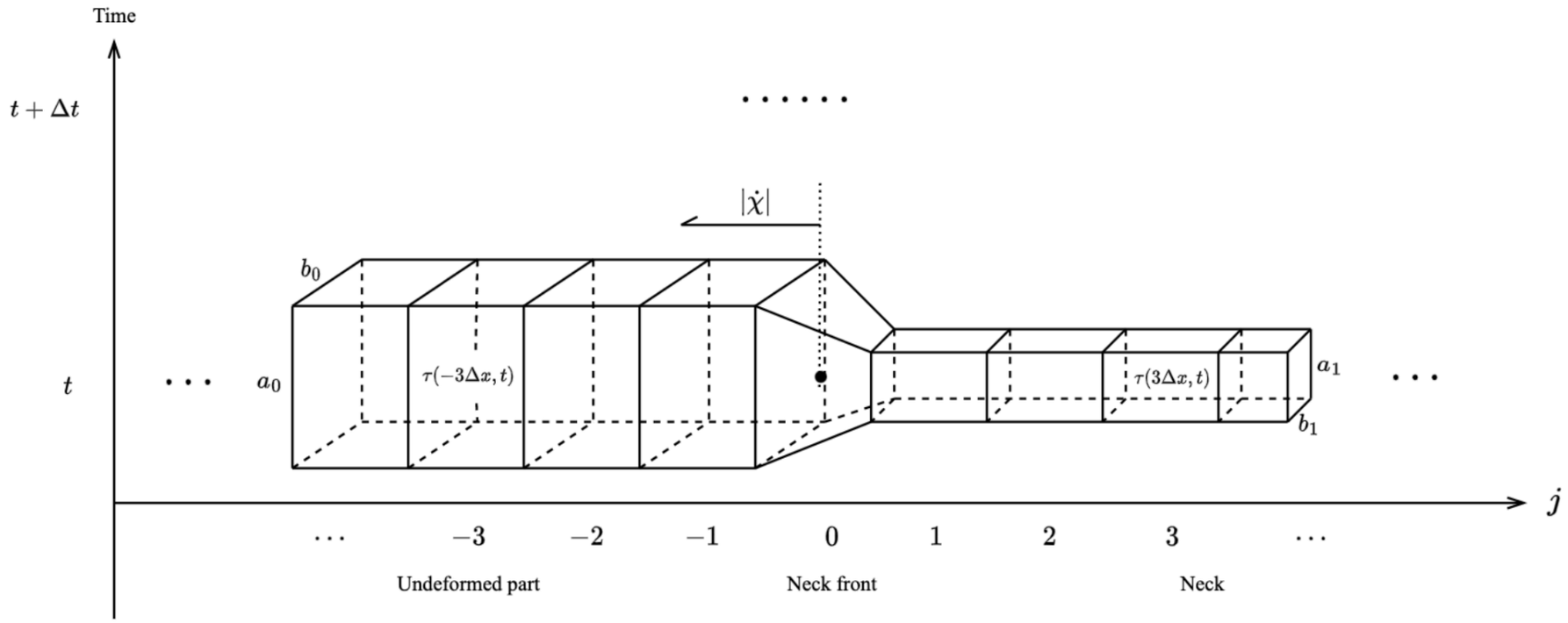
k_i $\left(\text{erg cm}^{-1}\ \text{s}^{-1}\ \text{K}^{-1}\right)$	E_i $\left(\text{g cm}^{-1}\ \text{s}^{-2}\right)$	$\rho\,\left(\text{g cm}^{-3}\right)$	c_p $\left(\text{erg g}^{-1}\ \text{K}^{-1}\right)$	λ
1.50×10^4	2.75×10^{10}	1.34	1.12×10^7	3.5
h_i $\left(\text{erg cm}^{-2}\ \text{s}^{-1}\ \text{K}^{-1}\right)$	α $\left(\text{cm}^3\right)$	$v_0\,\left(\text{cm s}^{-1}\right)$	ΔF $\left(\text{erg}\right)$	η
2.33×10^3	3.39×10^{-21}	2.25×10^{72}	8.78×10^{-12}	1

Partly Toda's, partly Nathan's

Fixed parameters

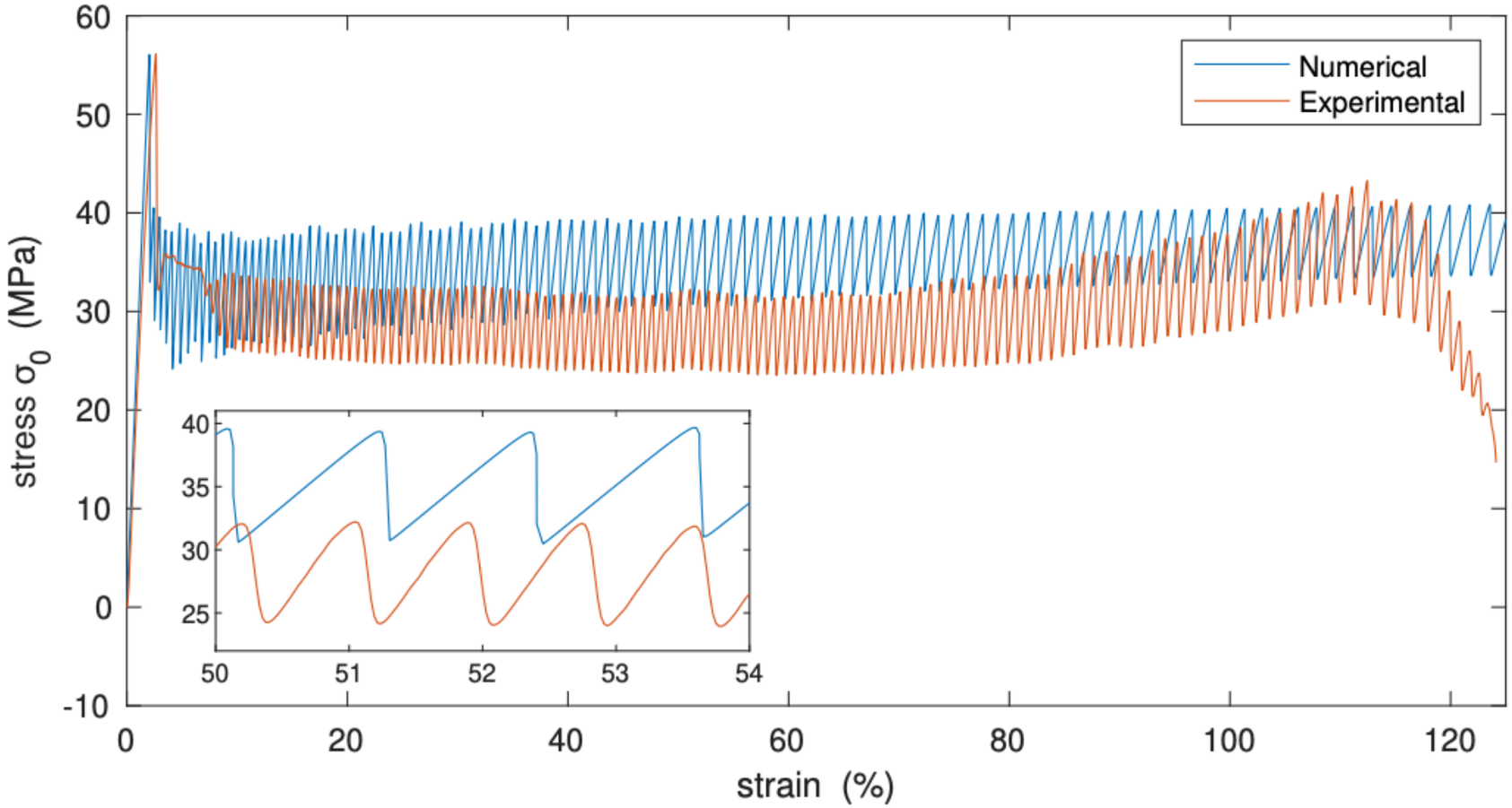
a_0 $\left(\text{cm}\right)$	b_0 $\left(\text{cm}\right)$	L_0 $\left(\text{cm}\right)$	k_B $\left(\text{g cm}^2\ \text{s}^{-2}\ \text{K}^{-1}\right)$	V $\left(\text{cm s}^{-1}\right)$	T_a $\left(\text{K}\right)$
0.657	0.008	6.965	1.3807×10^{-16}	$1/6$	293.15

Numerical computation



$$\Delta t = 0.005\text{s} \qquad \Delta x = \frac{1.5 - (-0.75)}{5000 - 1}$$

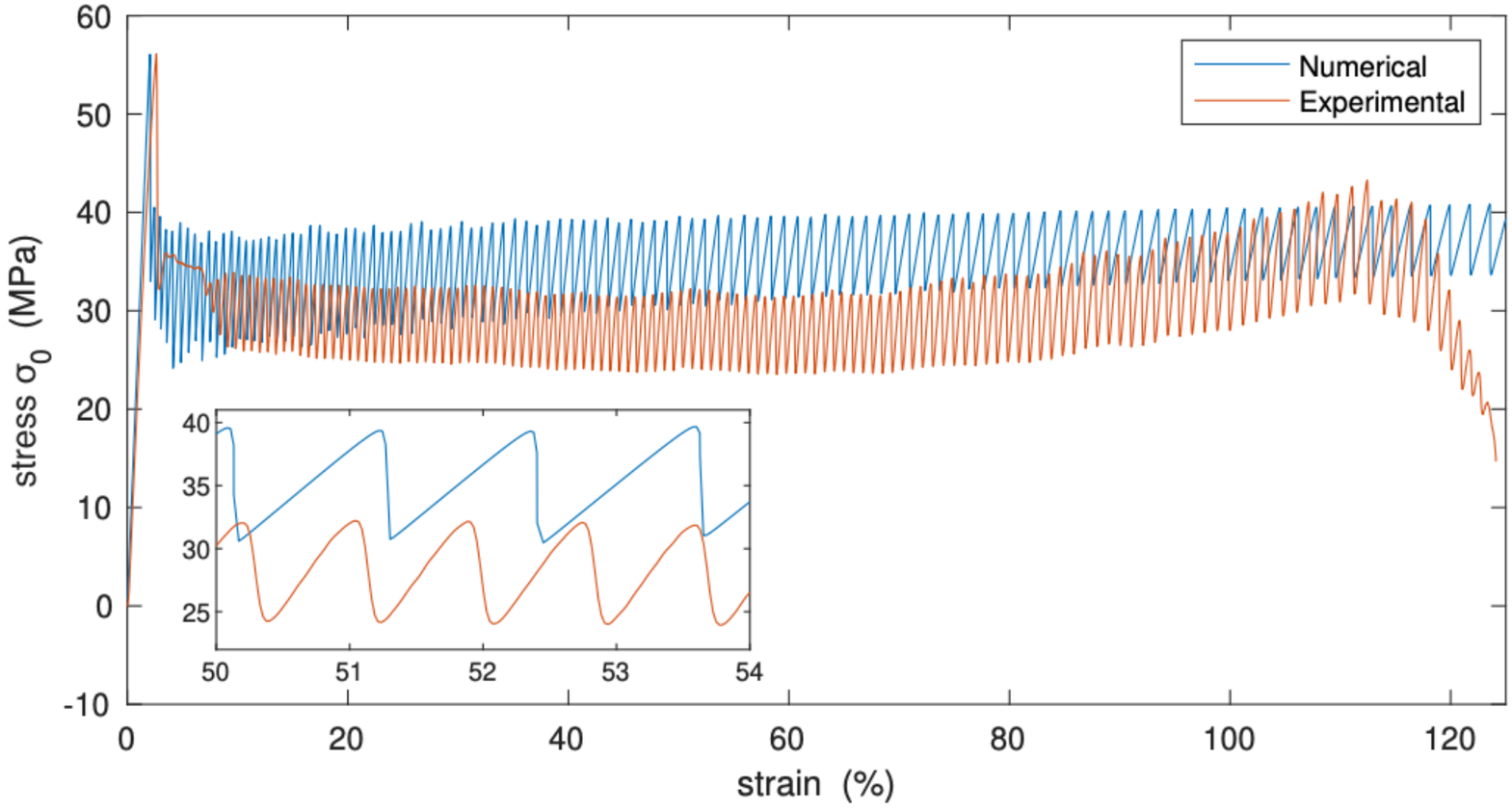
Main purpose



k_0, k_1 $\Delta F, \alpha$ h_0, h_1 v_0

Major problems

- The frequency
- The average stress
- The inclination
- The drop during oscillation
- Initial amplitude



Looking at single parameters

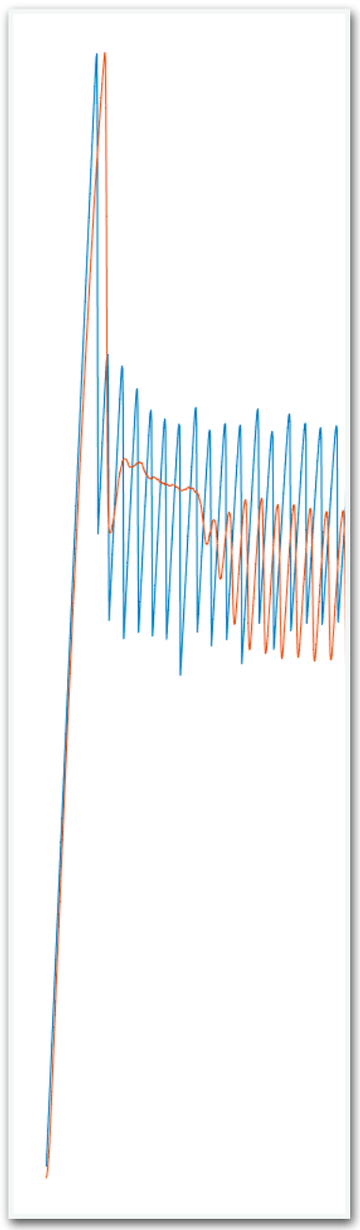
- Can we fit better ?
- How parameters affect stress-strain ?
- Any future experiments if possible ?

Young's modulus E_0

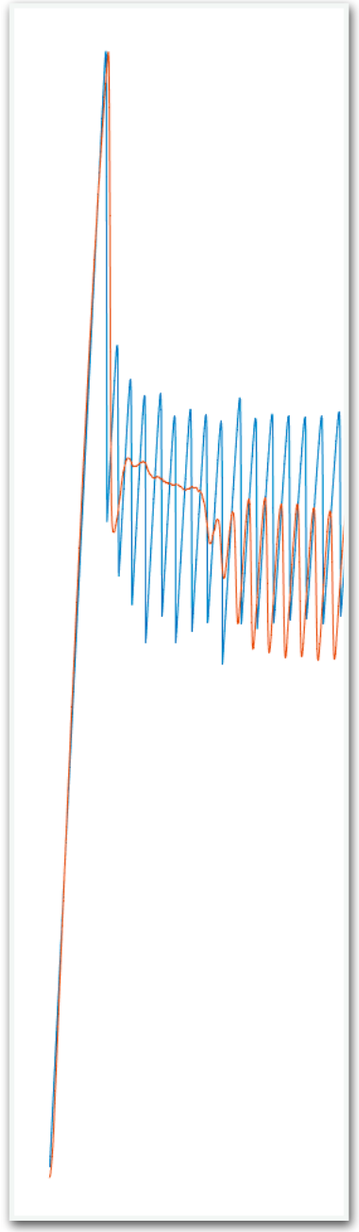
- E_0 can be directly fitted
- The value also has an effect on the stress-strain thereafter
- The experimental initial line is curved in a shape like:



$E_0 = 2.5 \times 10^{10}$

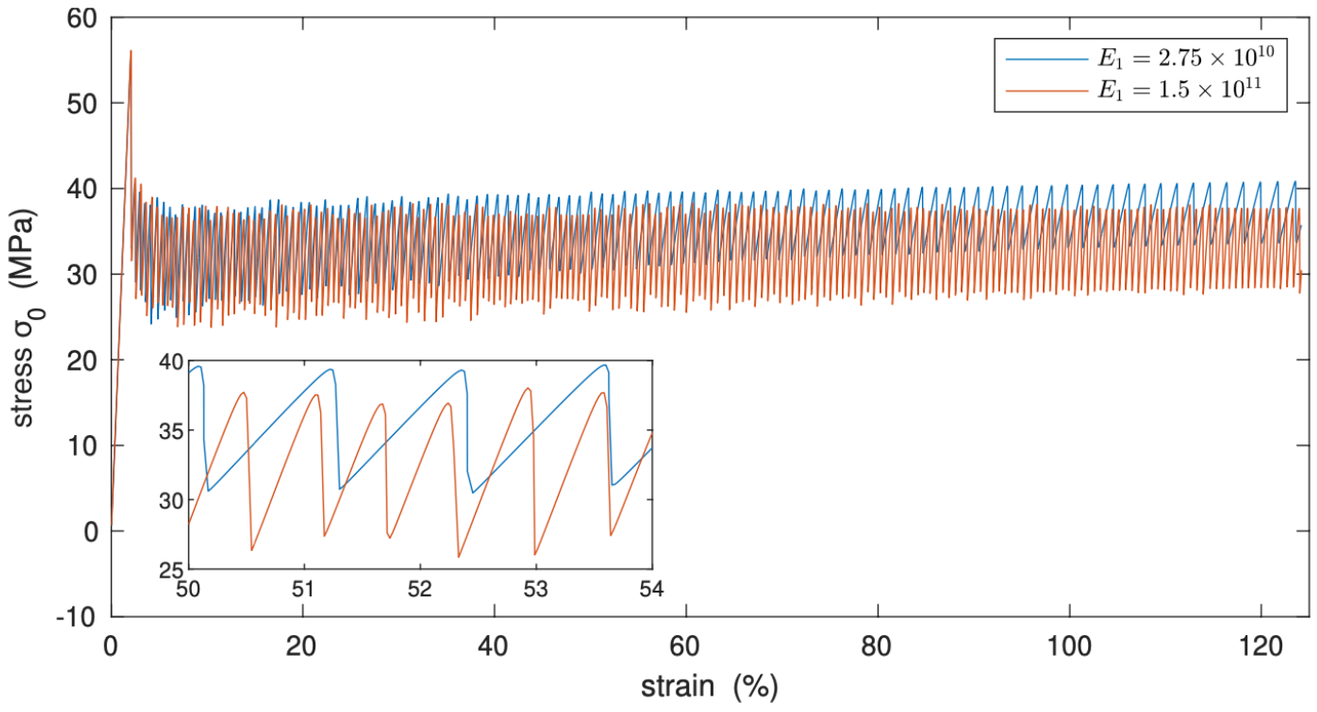


$E_0 = 2.25 \times 10^{10}$



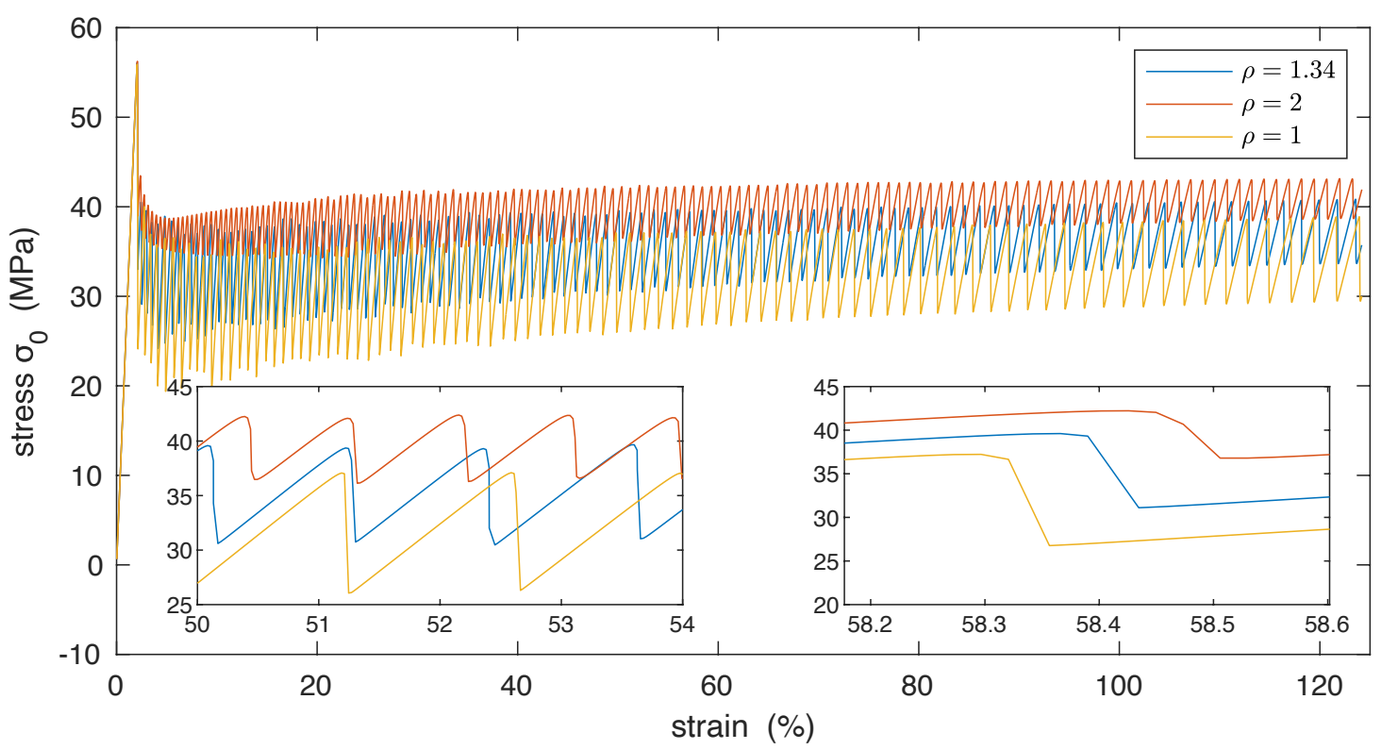
Young's modulus E_1 Somewhat sensitive

- E_1, as expected, affects the stress-strain after the initial line
- A larger value can adjust the inclination to decreasing
- With a large inclination, it's much higher in frequency, hard to trade off with other parameters. Better to get an experimental value if possible



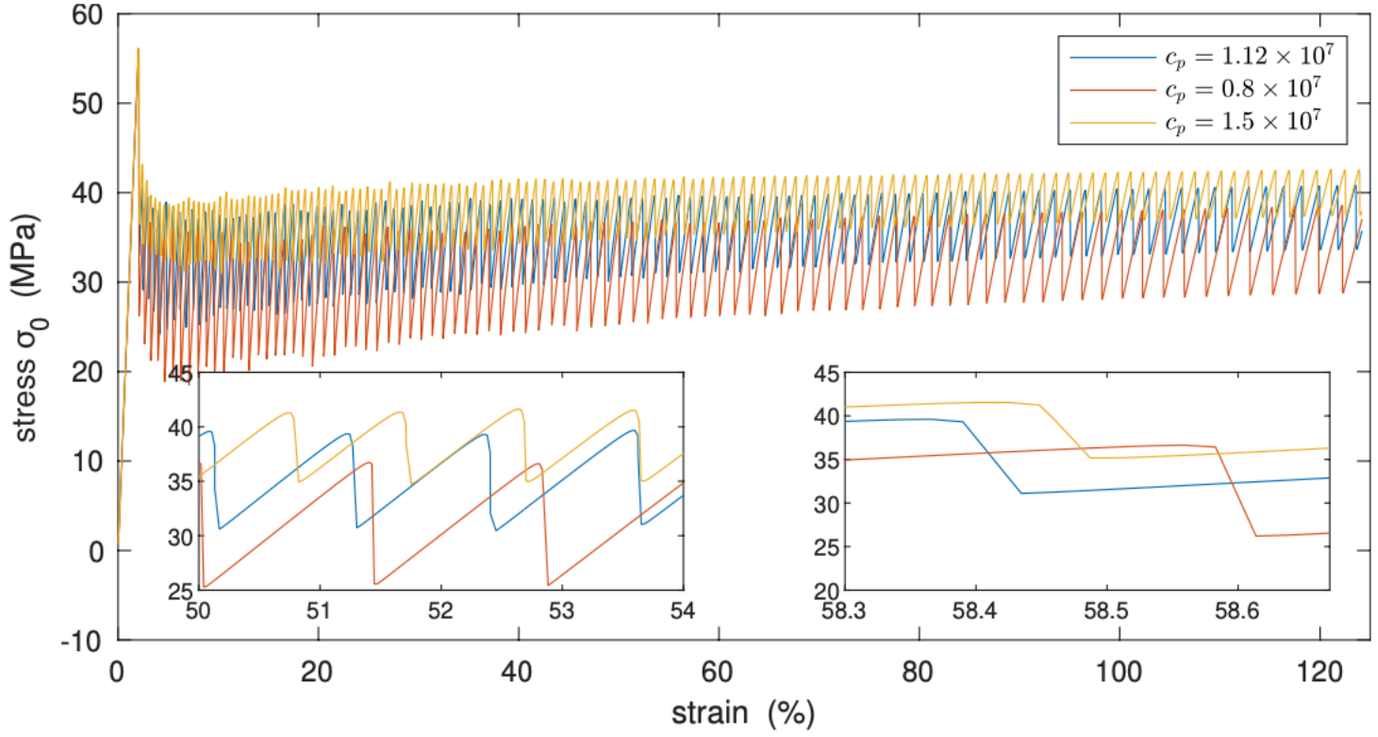
Sample density ρ Sensitive

- Smaller ρ means $\Longrightarrow$ smaller frequency & lower average stress & steeper drop
- Almost no effect at the yield point
- Seems different samples have different densities. Due to sensitivity, better to precisely measure the density



Heat capacity c_p Sensitive

- Similar effect as ρ
- Almost no effect at the yield point
- Due to complexity, Better to precisely measure the capacity



More on heat capacity c_p

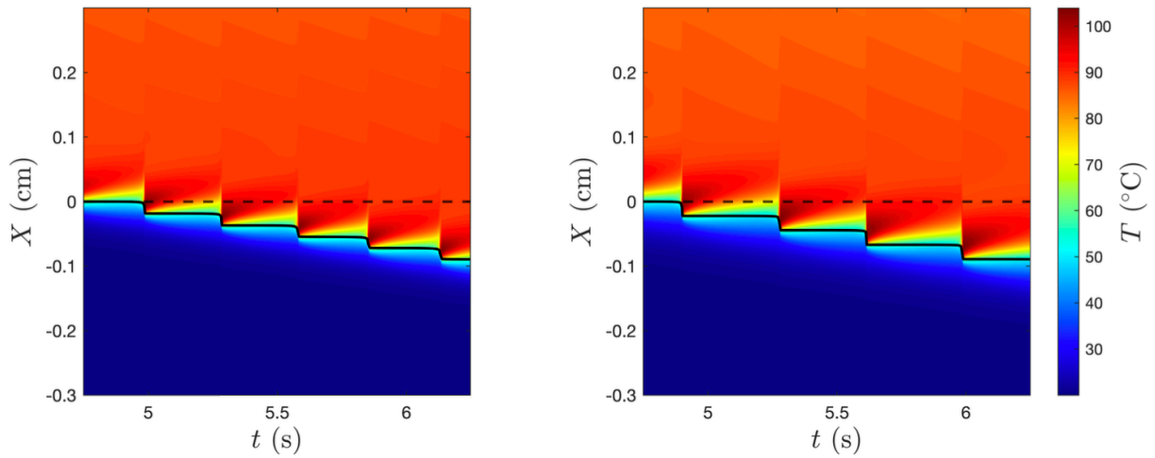


Figure 2.12: Temperature intensity plot with **Left:** $c_p = 1.12 \times 10^7$ erg g^{-1} K^{-1}, and **Right:** $c_p = 0.8 \times 10^7$ erg g^{-1} K^{-1}.

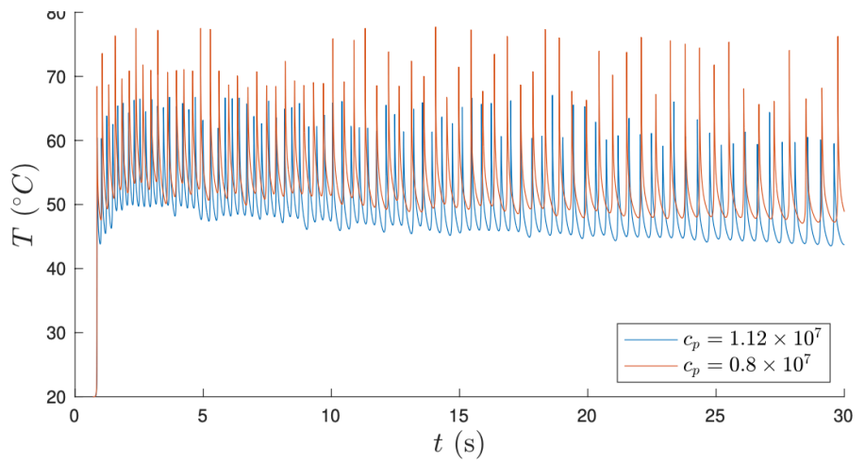
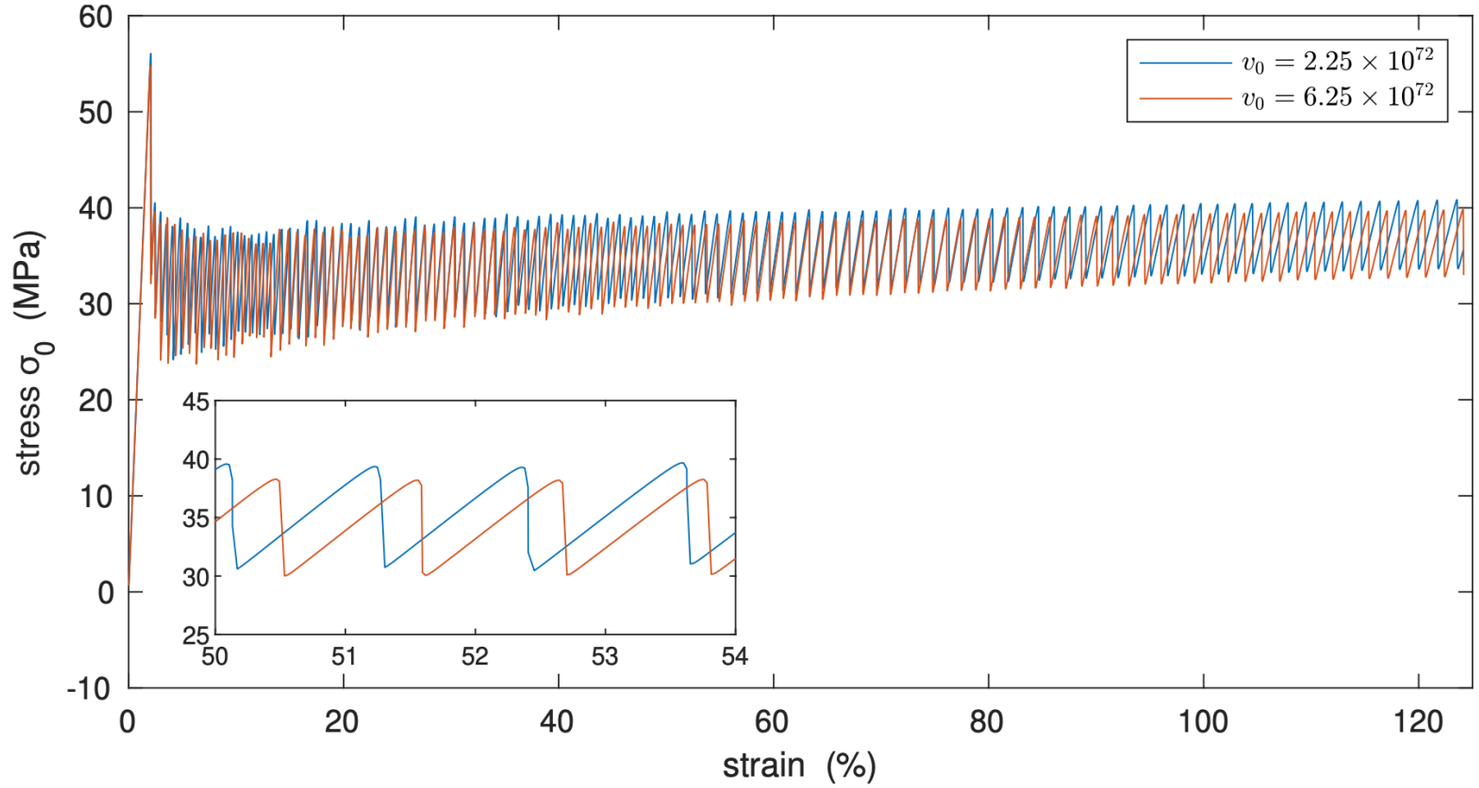


Figure 2.13: Approximated temperature at neck front against time. Comparing two different heat capacities.

Eyring's equation factor v_0

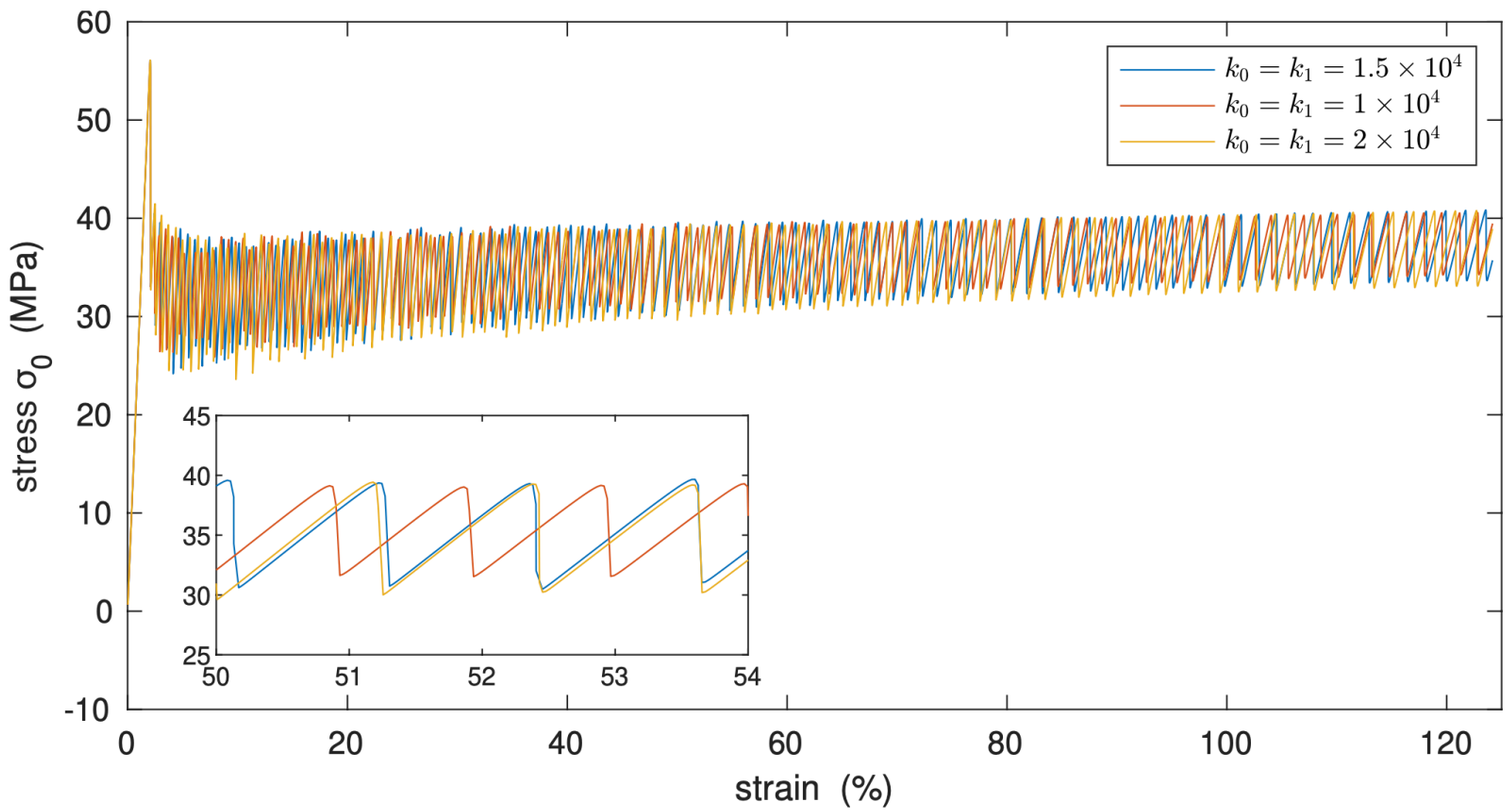
Insensitive

- Lifting the whole stress-strain curve
- Hard to measure
- Keep the value Toda used ?



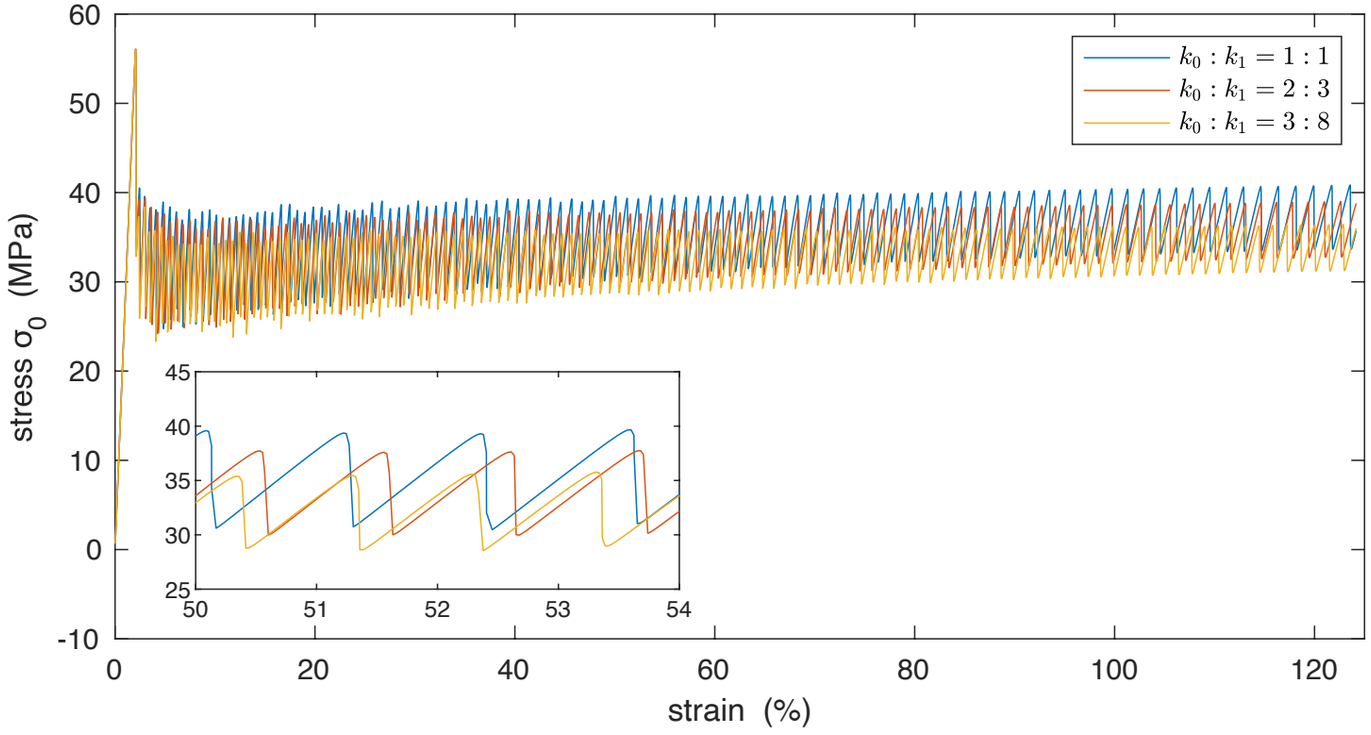
Diffusivity $k_0 = k_1$ Insensitive

- Hardly any effect compared to the magnitude of change, except phase change



Ratio $k_0 : k_1$ Somewhat sensitive

- Average stress suppressed, while preserving the yield point
- Amplitude suppressed through the whole oscillation
- Lead to frequency increase



More on diffusivity k_0, k_1

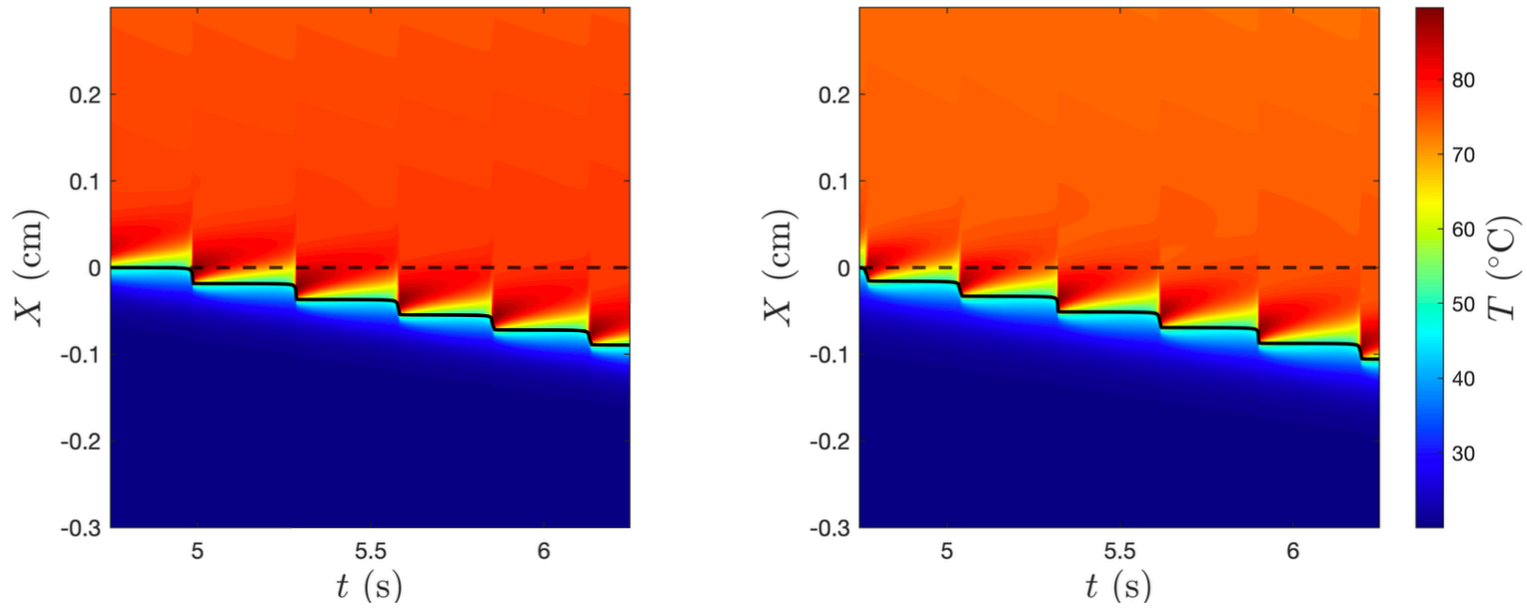
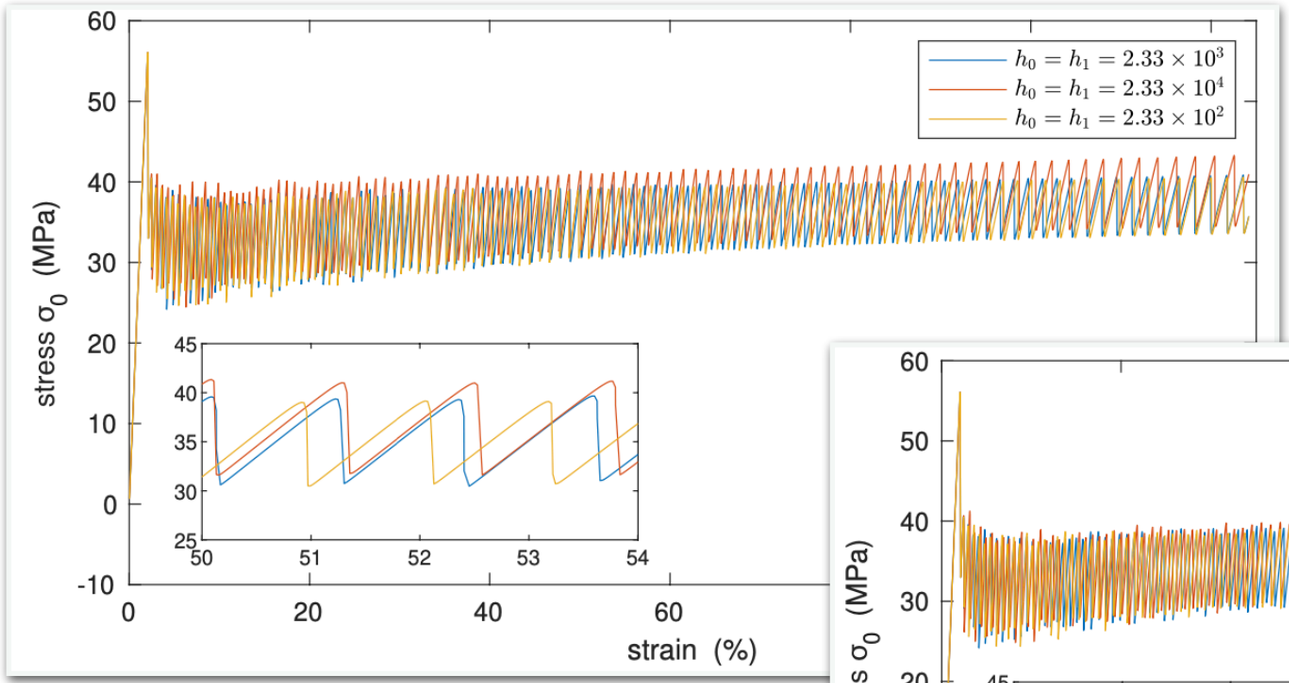


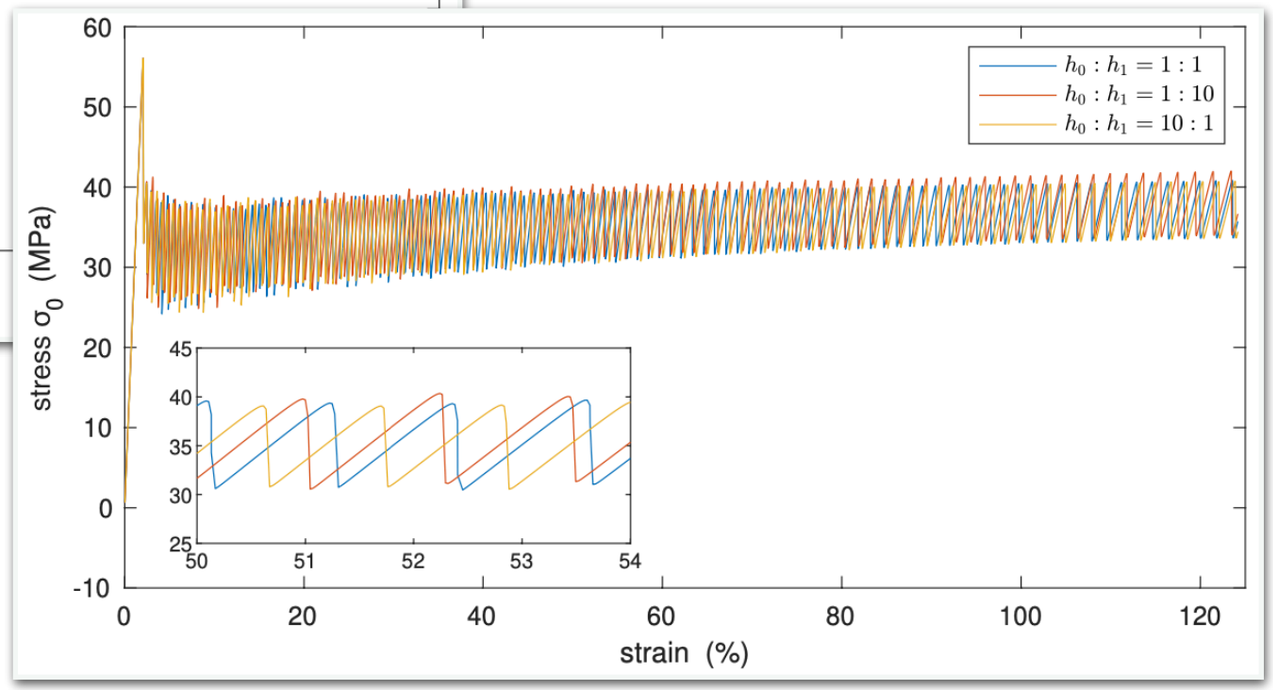
Figure 2.8: Temperature intensity plot with **Left:** $k_0 = k_1 = 1.5 \times 10^4$ erg cm^{-1} s^{-1}K^{-1}, and **Right:** $k_0 = k_1 = 2 \times 10^4$ erg cm^{-1} s^{-1}K^{-1}.

Heat dissipation to the air $h_0 : h_1$

The most insensitive

- Hardly any effect
- The ratio doesn't work this time





More on h_0, h_1

- The only way to fit h_0, h_1 seems to do experiments regarding temperature

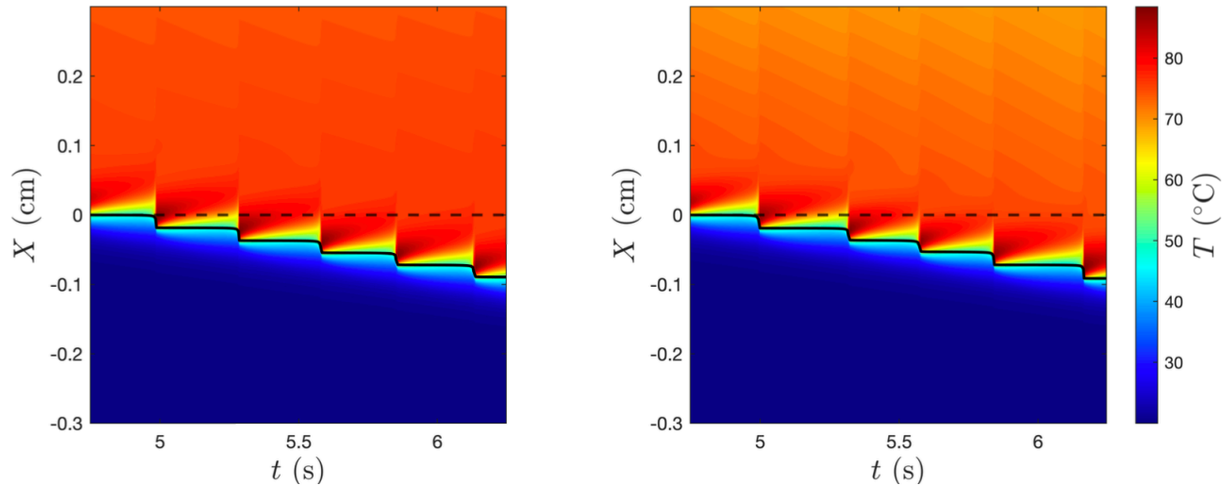


Figure 2.16: Temperature intensity plot with **Left:** $h_0 = h_1 = 2.33 \times 10^3 \left(\text{erg cm}^{-2}\ \text{s}^{-1}\ \text{K}^{-1}\right)$, and **Right:** $h_0 = h_1 = 8.33 \times 10^3 \left(\text{erg cm}^{-2}\ \text{s}^{-1}\ \text{K}^{-1}\right)$.

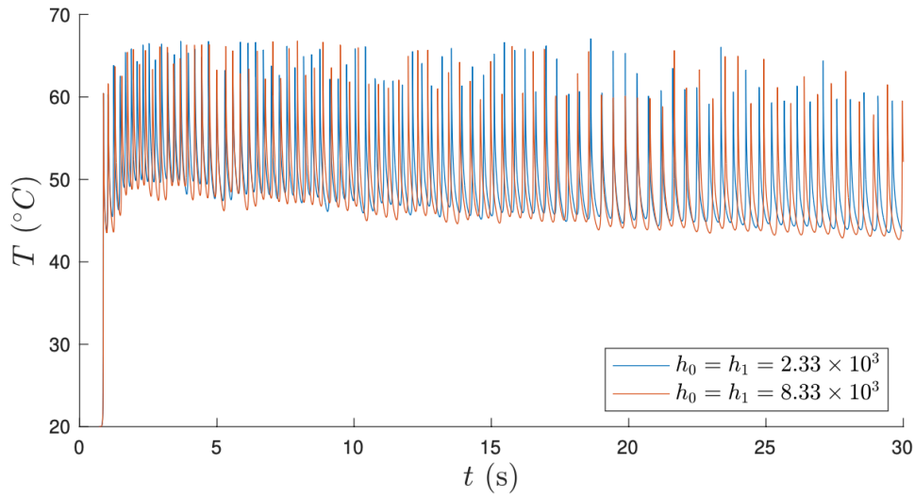
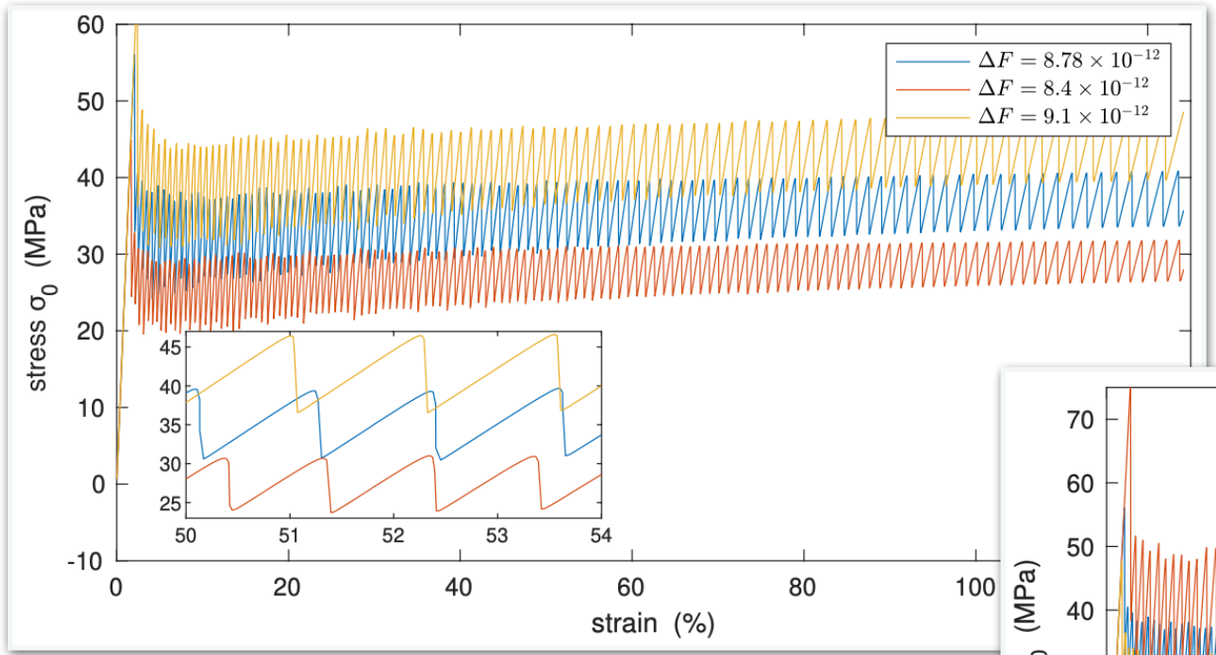


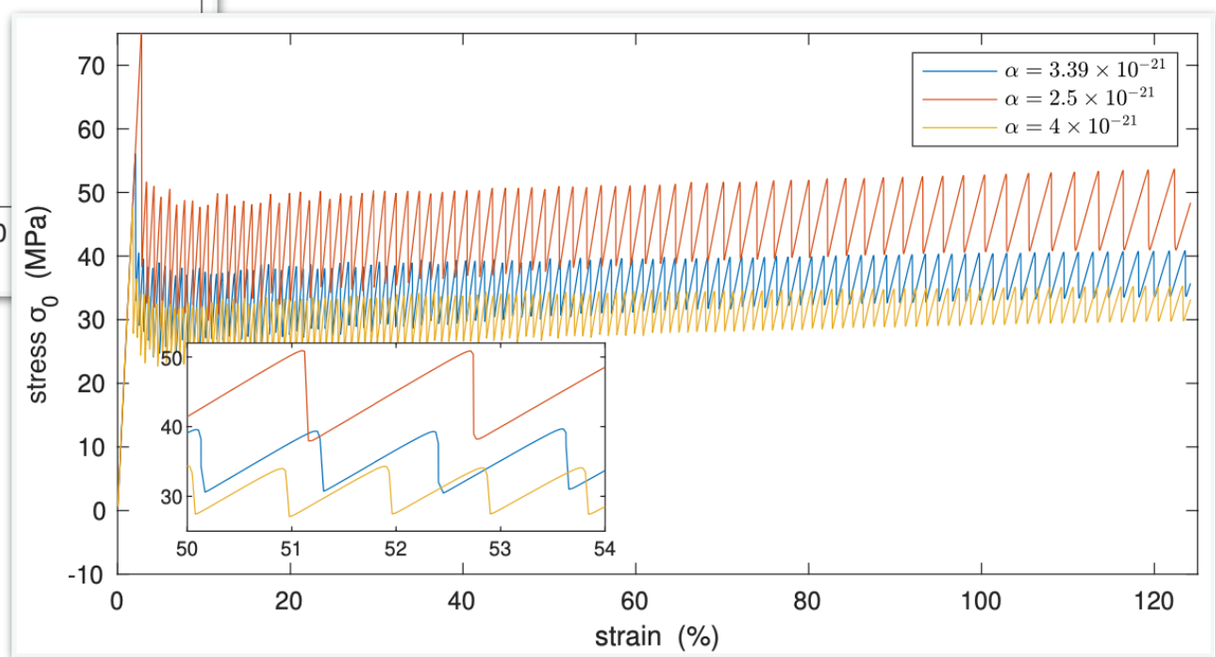
Figure 2.17: Temperature at neck front against time. Comparing two different sets of heat loss rate from sample to air.

Eyring's equation related $\Delta F, \alpha$

Sensitive

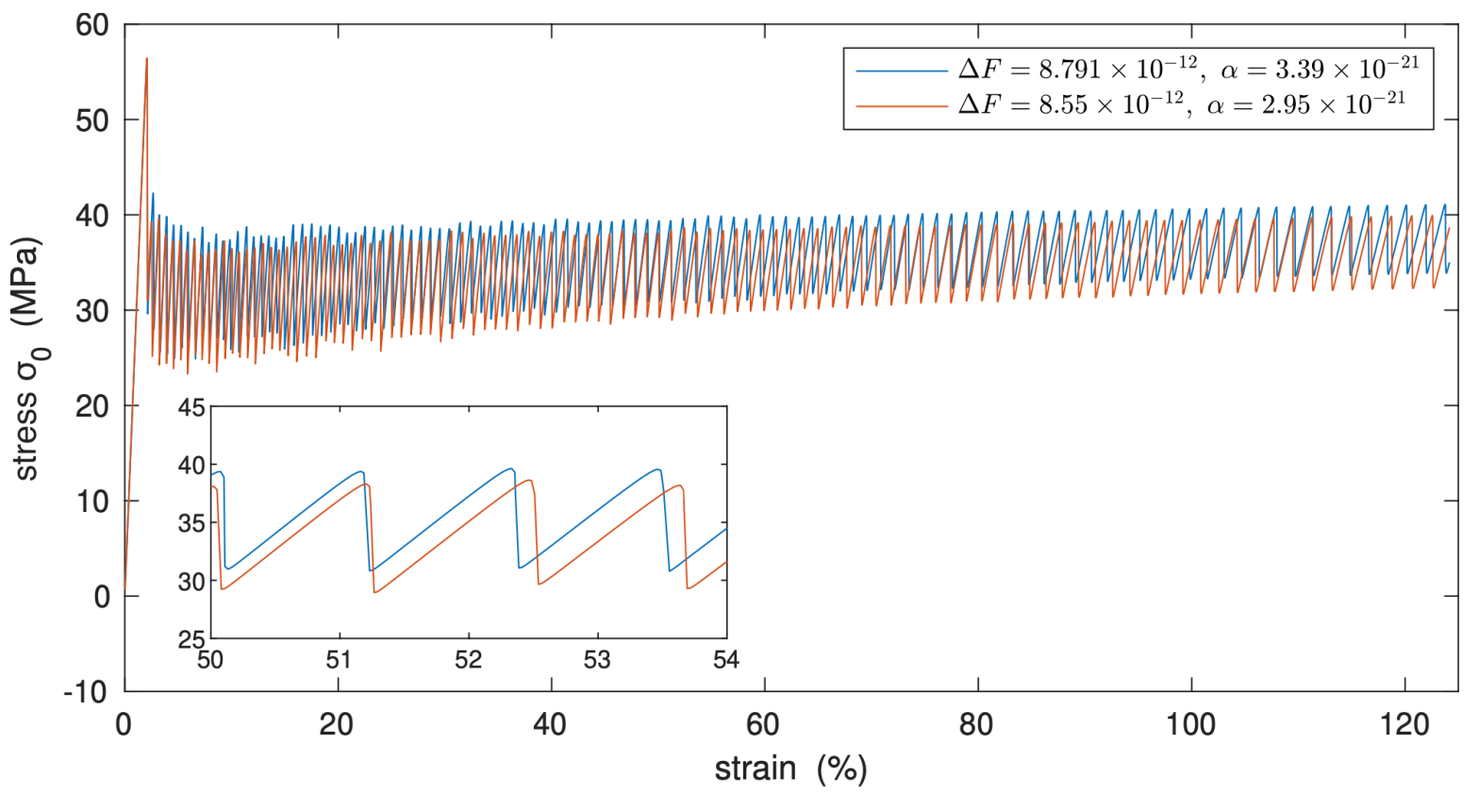
- Hardly any effect
- The ratio doesn't work this time





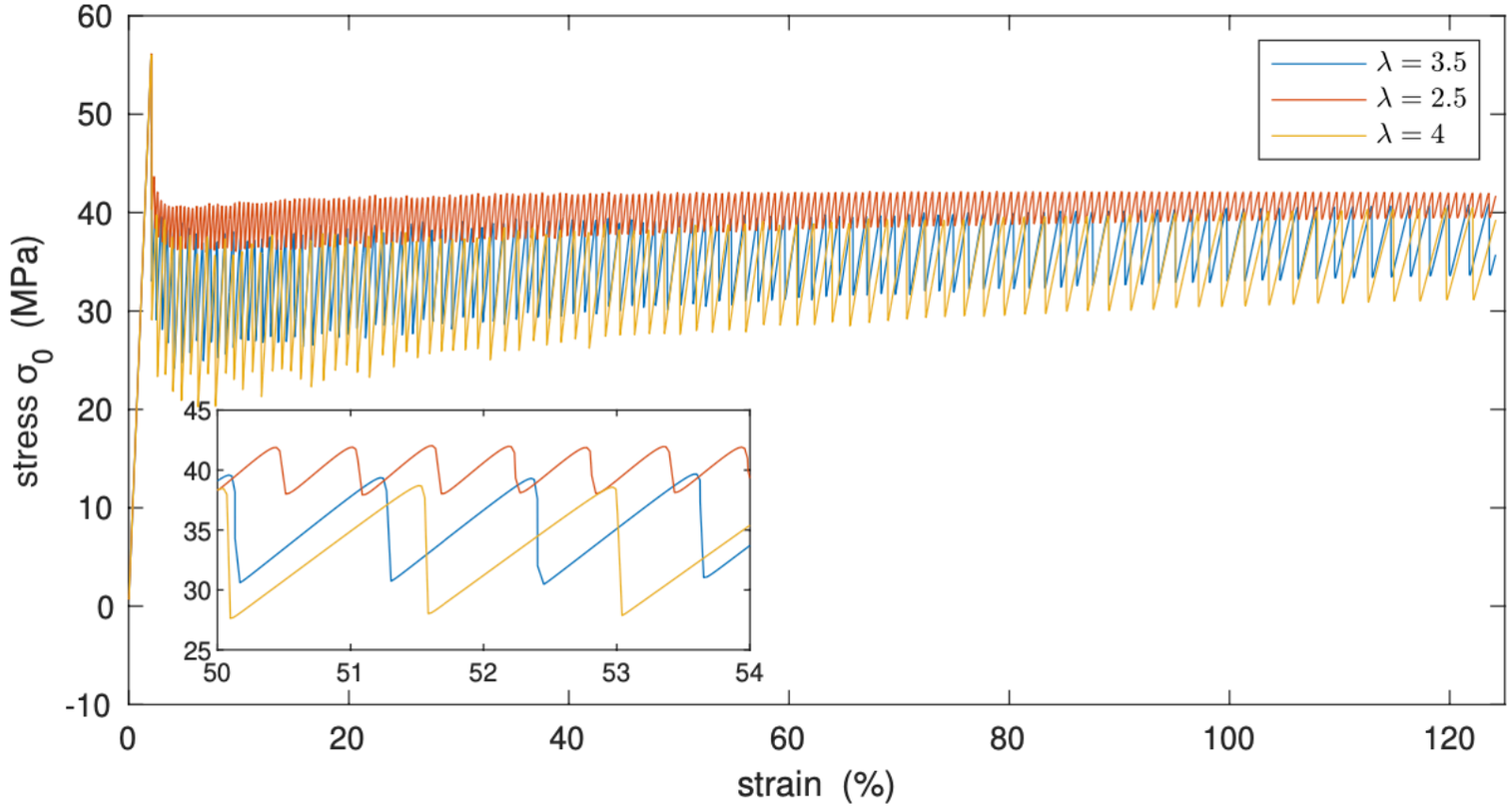
More on $\Delta F, \alpha$ Somewhat sensitive

- Adjusting the average stress while preserving the yield point
- Useful in tuning, it's often the case that we want to adjust the height of oscillation — and often it's too high, so we use a set of smaller $\Delta F, \alpha$
- There's an influence on frequency, but not obvious

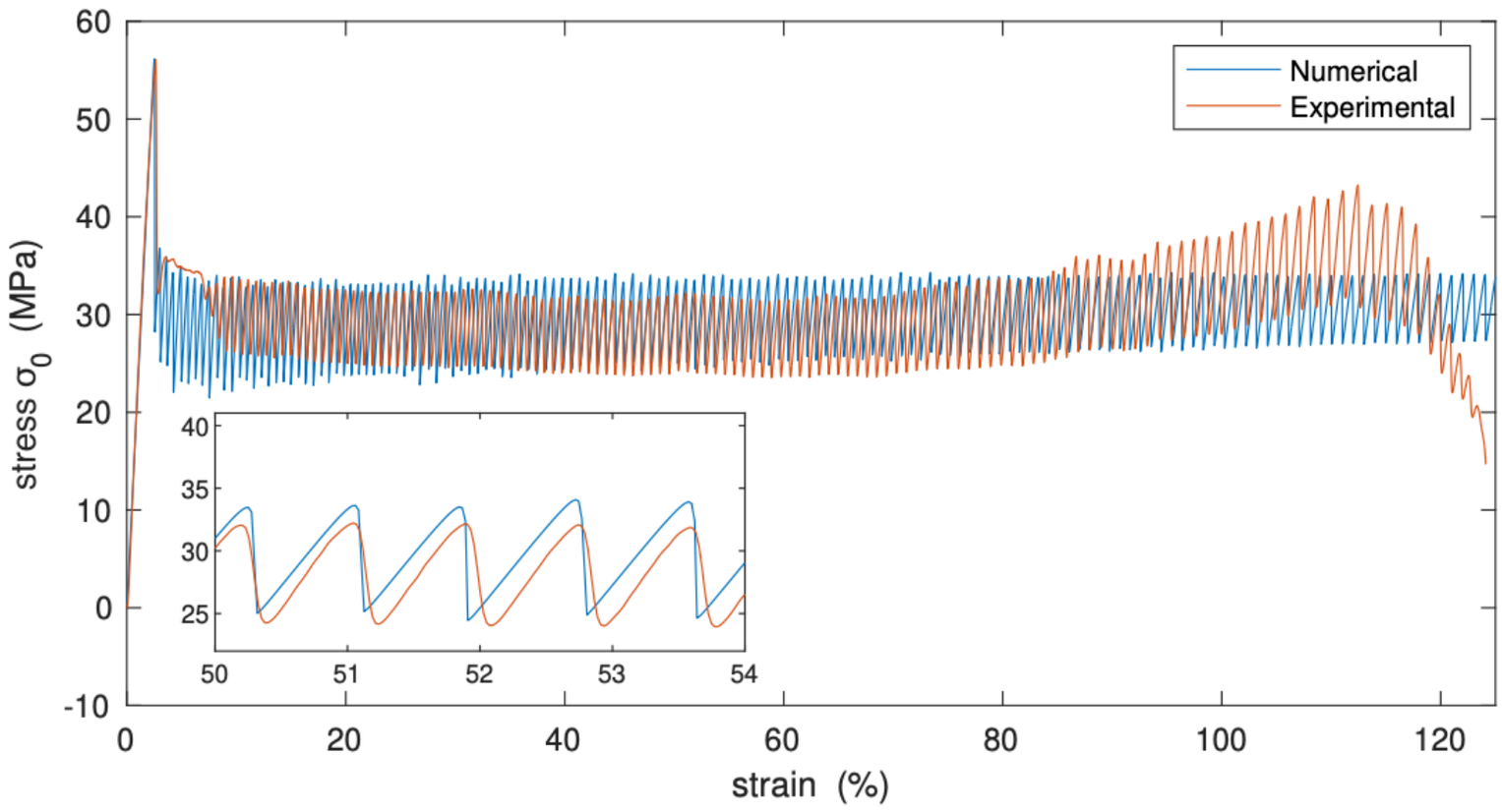


Draw ratio λ Sensitive

- One of the most sensitive parameters, can qualitatively affect the experiment, e.g. $\lambda = 2$
- I doubt that the ratio might be affected by many sample properties, e.g. initial length, width and thickness
- The parameter is so important that in my view it's better to include the measurement in experiments



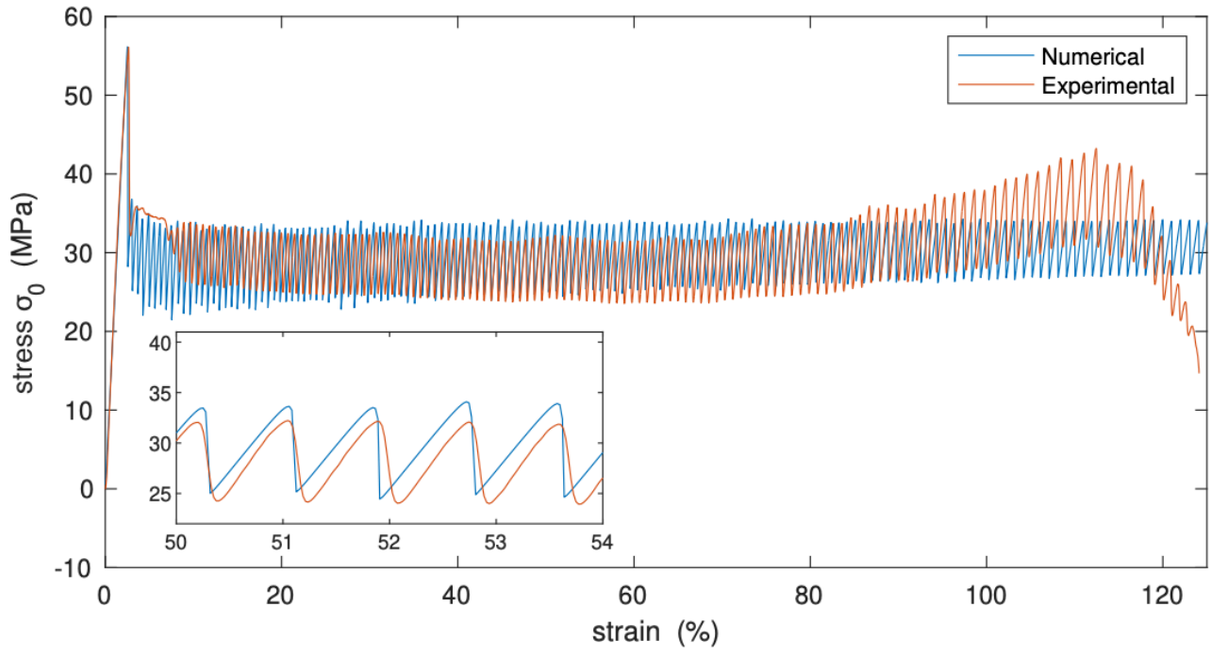
Results



k_0 $\left(\text{erg cm}^{-1}\text{ s}^{-1}\text{ K}^{-1}\right)$	k_1 $\left(\text{erg cm}^{-1}\text{ s}^{-1}\text{ K}^{-1}\right)$	E_0 $(\text{g cm}^{-1}\text{ s}^{-2})$	E_1 $(\text{g cm}^{-1}\text{ s}^{-2})$
1.50×10^4	3.70×10^4	2.25×10^{10}	6.1×10^{10}
h_i $\left(\text{erg cm}^{-2}\text{ s}^{-1}\text{ K}^{-1}\right)$	v_0 (cm s^{-1})	α (cm^3)	ΔF (erg)
2.33×10^3	2.25×10^{72}	2.95×10^{-21}	8.55×10^{-12}
ρ (g cm^{-1})	c_p $(\text{erg g}^{-1}\text{ K}^{-1})$	λ	η
1.38	1.04×10^7	3.5	1

a_0 (cm)	b_0 (cm)	L_0 (cm)	V (cm s^{-1})
0.657	0.008	6.965	1/6

Problem persists



- Yes: E_1 can adjust inclination But: Extreme value leads to extreme frequency
- Yes: Combining $\Delta F, \alpha$ can adjust the average stress
  But: All the techniques observed will worsen the amplitude at the beginning of oscillation
- Frequency is not really well fitted, although it's actually hard to fit everywhere since it varies
- Yes: c_p can make stress drop less steep But: The value then become twice the original one

Experiments if possible

(would be beneficial to have a fit with higher certainty)

Experiments	**Effect & Sensitivity**	**Importance of experiment**
Draw ratio λ	Qualitative effect; Quite sensitive.	Prior importance
Heat capacity c_p	Qualitative effect; Sensitive.	Great importance
Young's modulus E_1	The way to make stress decrease; Sensitive.	Important
Density ρ_0	Qualitatively effect; Sensitive.	Important
Neck front temperature	Beneficial to c_p, η	Optional
Thermal map	The only way h_0, h_1 can be fitted.	Optional

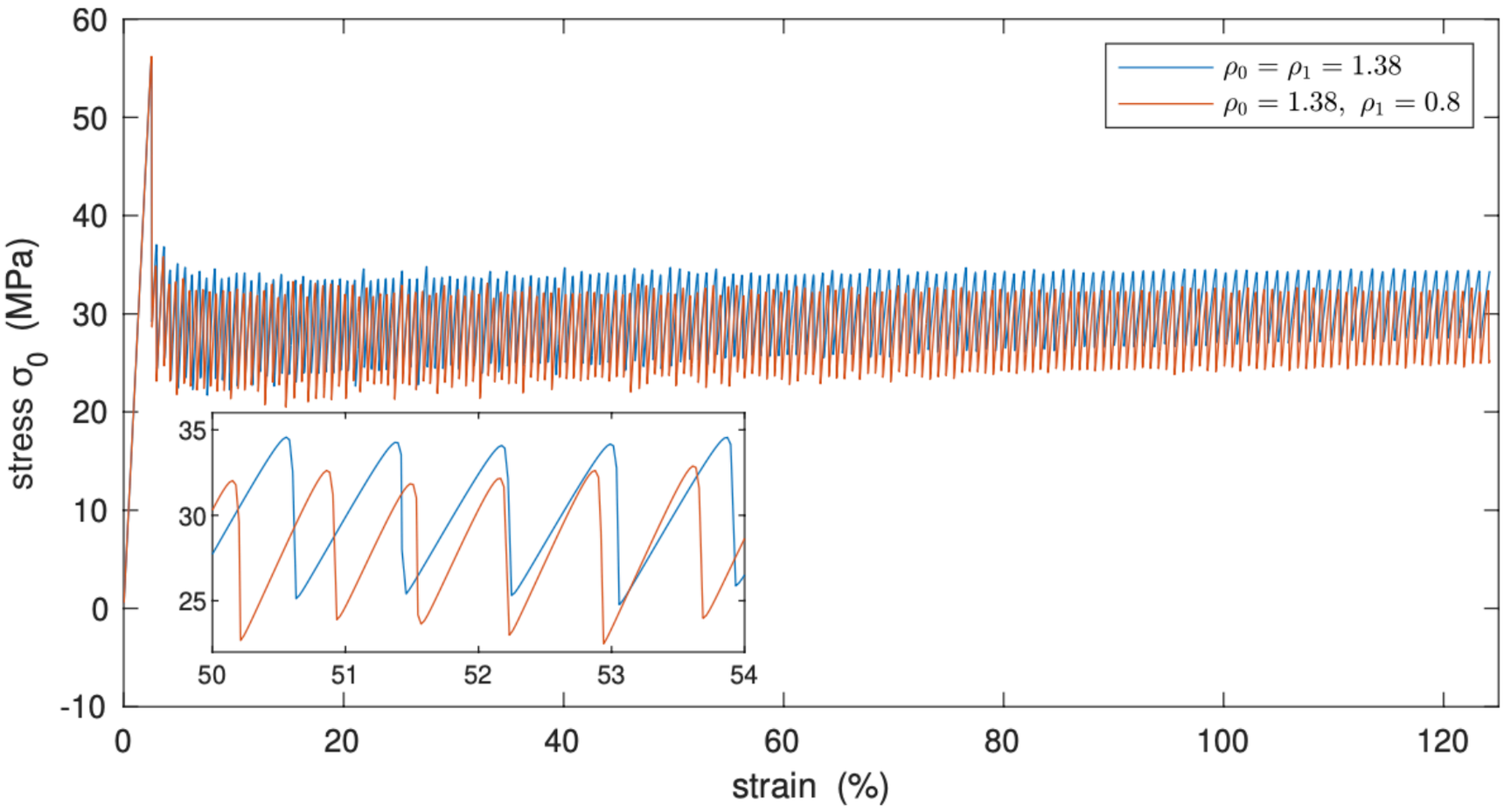
Table 5.1: Suggested experiments for future study.

Important tip:

It would be helpful if the width and thickness of the sample can be the same,
So that one can see if it's the length really determines the existence of stress oscillation.

" The most restrictive assumption in the generalised Toda's model might be the equality of density for undeformed and the neck "

Introducing the second density

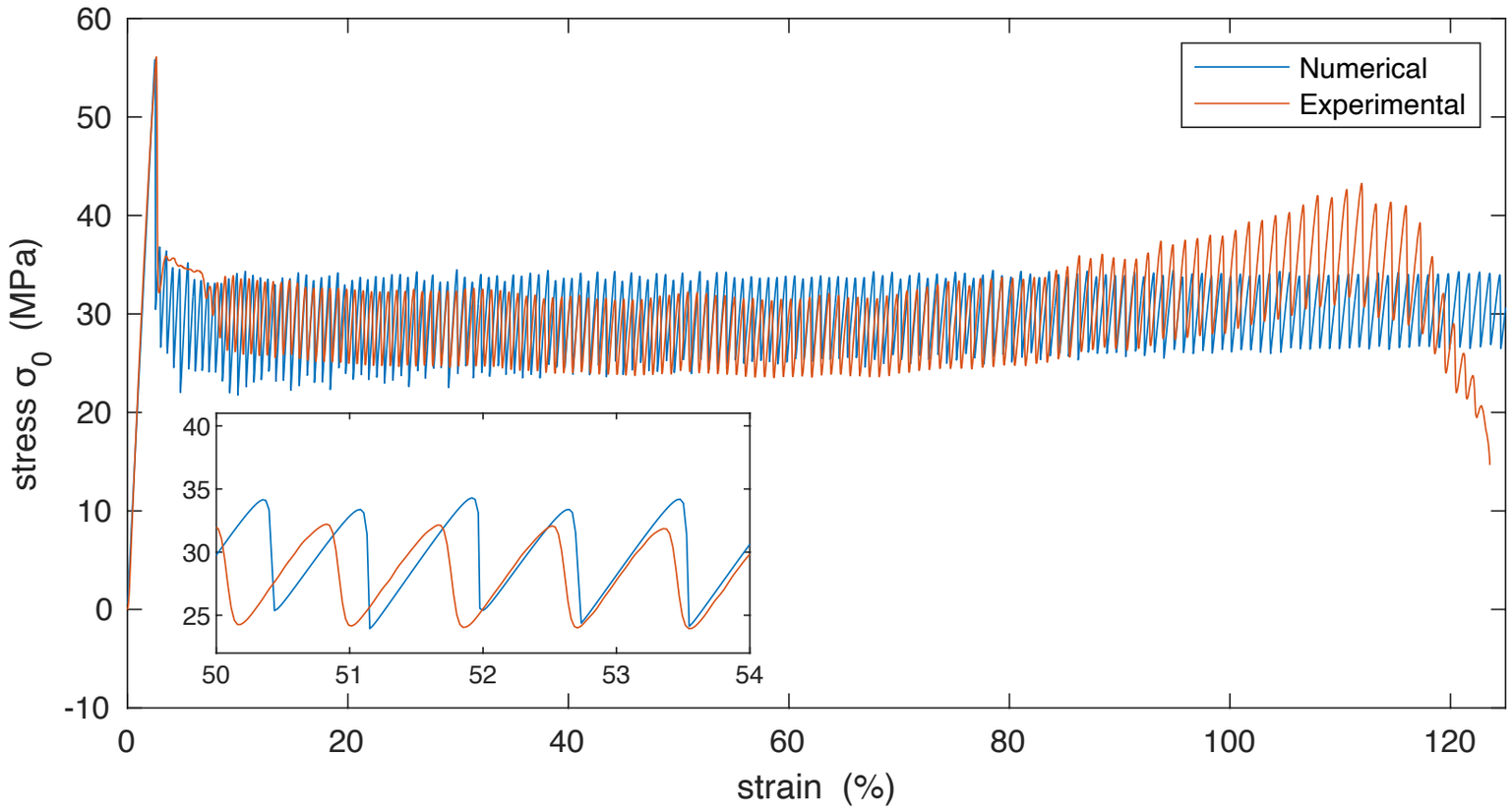


The effect is not as perfect as the expectation.
The average stress is decreased, but no gradual decrease pattern (inclination) is created.

And sadly the frequency increases as a consequence — it's actually not intuitive,
Since the frequency should be smaller for small density as mentioned.

We've known the second density gives another way to
adjust the average stress while preserving the yield point.

Another potential advantage




```
 struct('v_0',2.25e72,'T_a',293.15,'rho_0',1.5,'rho_1',1,'c_p',1.2e7,'h_0',2.33e3,
'k_0',1.5e4,'k_1',3.5e4,'k_B',1.3807e-16,'dF',8.45e-12,'alpha',2.8e-21,'eta',0.9,'
'E_0',2.25e10,'E_1',6.25e10,'lambda',4,'a_1',0.657,'b_1',0.008,'L_0',6.965); %simp
```

With a higher ρ_0 (a bit higher than usual) while smaller ρ_1,
the amplitude at the beginning of oscillation can be suppressed.